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ABSTRACT

Current thinking suggests that solving brainteasing puzzles uses the same critical thinking skills needed to solve difficult math, science, and business problems. This book is a non-intimidating exploration of the wonderful powers of the mind with an emphasis on the joy of thinking and learning. It contains 100 puzzles, presented in order of mathematical difficulty, that were specifically designed to be both fun and challenging and to promote the idea that becoming a better problem solver can increase the flexibility of one's mind. Answer keys are included. (JRH)

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BRAIN STRETCHERS

BOOK 3 – ADVANCED

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CRITICAL THINKING BOOKS & SOFTWARE

TERRY H. STICKELS

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BRAIN STRETCHERS

Book 3 – Advanced

Terry H. Stickels

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Introduction

A good puzzle can be both fun and challenging; it can also be educational. Current thinking suggests that solving these brainteasing puzzles uses the same thinking skills needed to solve difficult math, science, and business problems. Although intentionally designed to be lighthearted, "brain stretchers" demand the same discipline required to solve any difficult problem: drawing valid conclusions based on logic and evidence. Some of the more demanding puzzles are direct descendants of problems from mathematics and science, fields in which we need the brightest and most flexible minds.

Progressive companies are now recognizing the benefits of a work force of critical and creative thinkers. Author and consultant Michael Michalko of Rochester is more aware of this fact than are most people. Michalko's best-selling book *Thinkertoys* has been labeled by numerous business executives as the "business book of the nineties." What sets it apart from other business books? It is Michalko's use of "thinkertoys"—activities requiring critical and creative thinking. These activities steer the brain into discovering new ideas and offer an approach to problem solving unlike any other.

The idea of using puzzles to increase our minds' flexibility is not a new one. Let's take a look at what some noted thinkers have said about the relationship between critical thinking puzzles and learning.

Charles Sanders Peirce (1839–1914) is considered by many to be America's greatest philosopher. Martin Gardner, puzzle creator extraordinaire, writes about Peirce in his book *Fractal Music, Hypercards and More...*,"He was convinced that the methods then in use [for teaching in the schools] produced only poor students. The manuscripts of his three unpublished textbooks are filled with novel ways of using puzzles, games and toys for introducing mathematics concepts....He delighted in asking teachers to let him instruct a group of youngsters who detested mathematics and seemed incapable of learning it. He records that in one case, after about 10 lessons, two of his students led the school."

In Germany during the 1920s, a group of psychologists from the Gestalt school, led by Karl Duncker, initiated research into developing thinking skills by use of brainteasers. Duncker's work on "reorganization" and "solution trees" is considered to be part of the foundation for studying the intricate workings of the human mind.

You may remember physicist Richard Feynman as the man who put the rubber O-rings into the glass of ice water to demonstrate to Congress why the Challenger exploded. But Feynman's legend began long before those hearings. There are few who would disagree that Feynman had one of the most brilliant minds of the 20th century—a mind that was literally obsessed with solving and creating puzzles and brainteasers.

Feynman saw every problem as a potential brainteaser. He deplored traditional methods of solving problems and was always seeking ways to simplify these methods. He even developed his own mathematical symbols, finding

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them to be more concise and less bogged down than traditional notation.

Feynman believed that you should try to "understand simple things in terms of other ideas" and that tackling a problem head-on was "second best." He thought that the mind should consider misdirection as much as forward direction.

Brain Stretchers Book 3 — Advanced is a nonintimidating exploration of the wonderful powers of the mind—with an emphasis on the joy of thinking and learning.

The puzzles are presented in order of mathematical difficulty; however, what is easy for one person may be difficult for another and vice versa. They were specifically designed to be both fun and challenging and to promote the idea that becoming a better problem solver can be a rewarding experience.

The following suggestions may help as you attempt to solve the brain stretchers:

1. Solutions can often be reached just as easily by eliminating possible choices.

   "It is an old maxim of mine that when you have excluded the impossible, whatever remains, however improbable, must be the truth."

   —Sherlock Holmes,
   The Beryl Coronet

2. Don't be afraid to take a totally different approach—even if it seems absurd.

   "By indirections, find directions out."

   —Shakespeare

3. Make a written legend of the data, facts, and information you're given. Put it into some semblance of order that's easy for you to manipulate. The mere act of writing or typing can often trigger the mind into action.

4. Break components of a difficult problem down into simpler forms. Use easier, similar examples for reference.

   "There can be no mystery in a result you have discovered for yourself."

   —W.W. Sawyer
   20th-century Author

5. Look for constant relationships and patterns.

6. Turn the problem inside out, upside down, and sideways. Spin it, twist it, stretch and break it. Play with it (both literally and figuratively). Put the problem into different frames of reference to view it.

7. After you've reached a solution, plug your answer back into the problem to see if it fits all the parameters you were asked to consider.

8. Skepticism vs. cynicism. Always be wary, but leave the cynicism to the columnists and commentators. Healthy, respectful skepticism is a must for growth and success.

   "In order to seek truth, it is necessary once in the course of our life to doubt as far as possible all things."

   —René Descartes
   French Mathematician
   and Philosopher

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9. Don't be afraid to fail—don't be afraid to say, "I was wrong." Remember: it is when you are wrong that people want to see how you'll react, not when you are right.

   "Mistakes open the door to insight."

   —T. H. Stickels

10. When all else fails
    - Get away from the problem temporarily.
    - Talk out the problem with someone else (it worked for Sherlock).

Finally, I'd like to leave you with several ideas to consider in your adventure of becoming a better problem solver:

1. When closing in on a solution, keep your emotions and personal prejudices out of your solutions and check your ego at the front door.
2. Nothing replaces good old-fashioned hard work.
3. A little humor never hurt any good problem solver.
4. Challenge yourself—overcome your fears. It is seldom the intellectual roadblocks that stop us; instead, it is the lack of courage in our convictions.
5. Read, observe, listen.
6. Mentally create new problems—keep covering new bases.
7. The amount of time spent on reaching a solution has nothing to do with whether or not you are a good problem solver.
8. Question everything, including what you've read here. There might be an even better avenue for you to take—one you've discovered on your own.

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Puzzles

1. What is the next figure in the following series?

W T M T E S ?

2. Below are three views of a solid object. See if you can draw either side view.

Top

Front

Back

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3. Bill and Bob have Johnson and Mason for last names, but I can never remember if it is Bill Johnson and Bob Mason or Bill Mason and Bob Johnson. I do know that two of the following statements are false. Given that information, what are the correct full names of each?

- Bill's last name is Johnson
- Bill's last name is Mason
- Bob's last name is Mason

4. What is the next number in the following series?

2 4 8 14 22 32 ?

8

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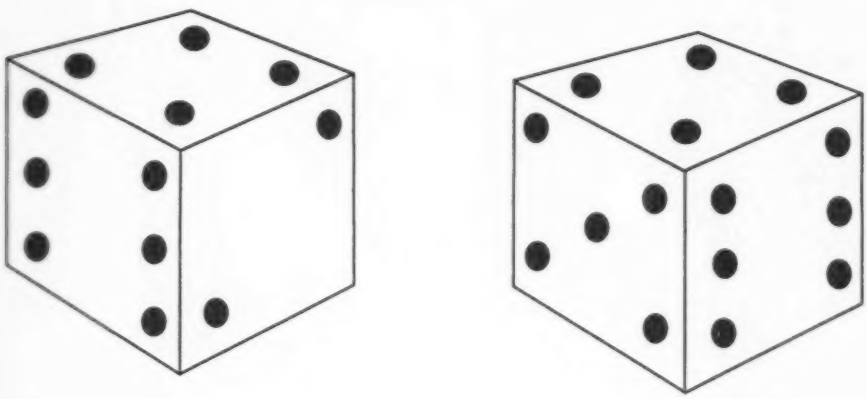
5. What is the next number in the following series?

HINT: Just because numbers are used doesn't always mean the relationship between them is mathematical. These numbers spell a word.

16 21 26 26 12 5 ?

6. The sum of the dots on opposite faces of one die is always the same. In the picture below are two normal dice. How many dots are on the face opposite the face with four dots?



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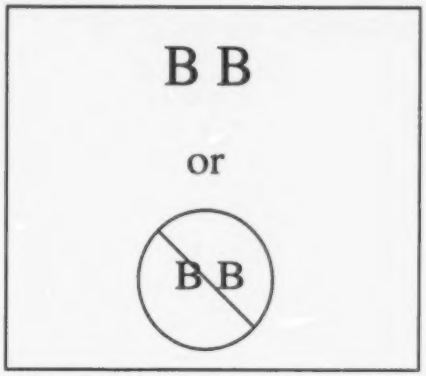
7. Following are sixteen "framefrases" where you are to determine a common phrase or song.

   Here's an example:



The answer for this framefrase is "To be or not to be."

Framefrases can be a lot of fun. When you're done solving these, make up some of your own to share with family and friends.
WARNING: These puzzles can become habit-forming.

a.

10

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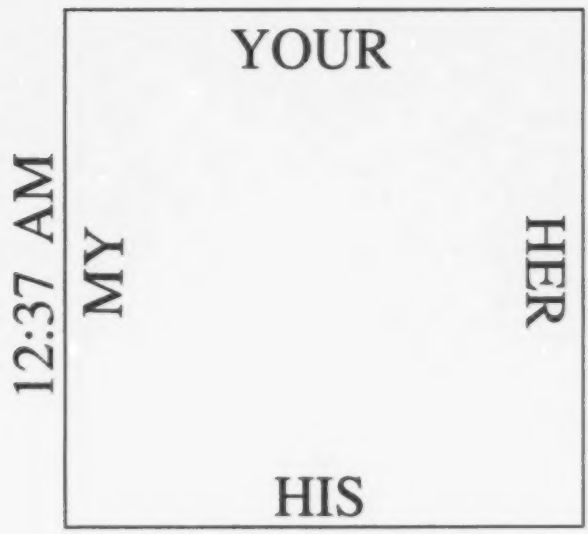
b.



c.

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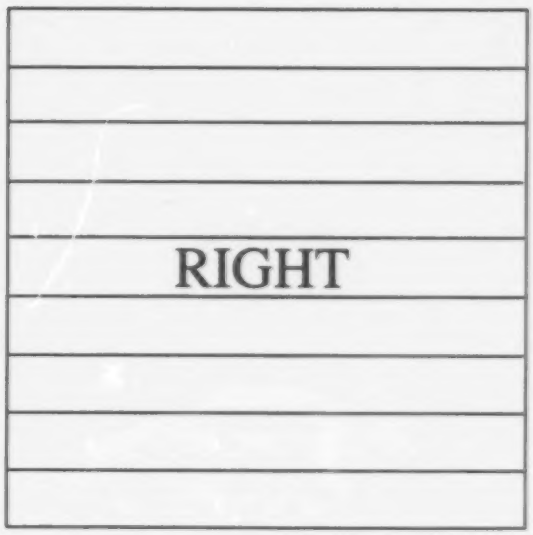
d.



e.

12

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f.

g.

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h.

i.

14

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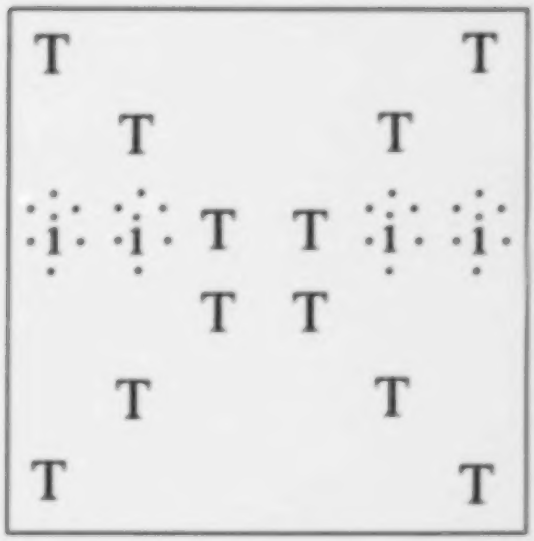
j.



k.

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l.

m.

16

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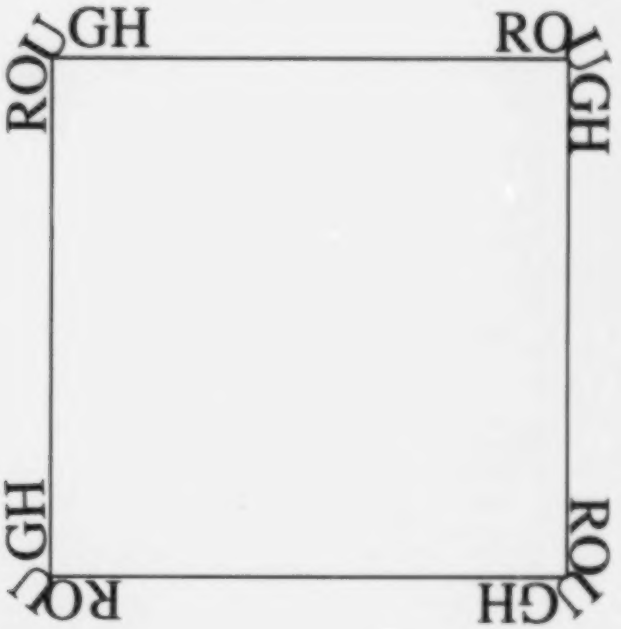
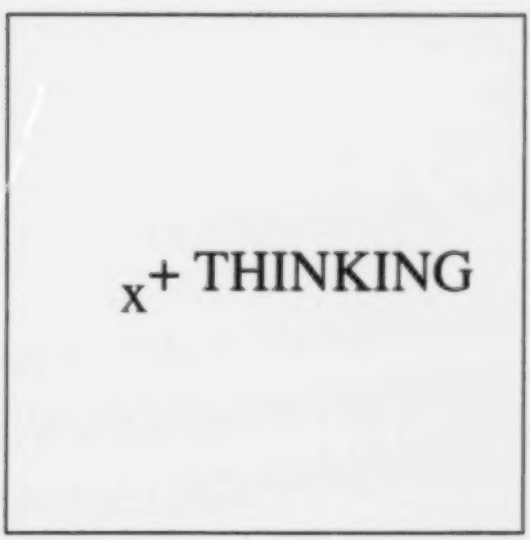
n.



o.



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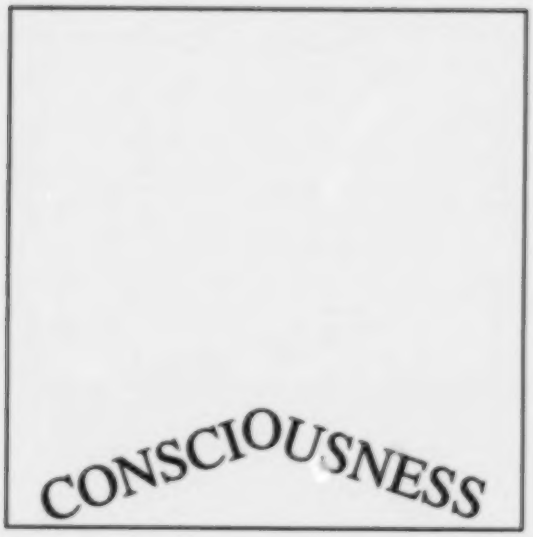
p.



8. A bag contains two white balls and one red ball. What are the chances that you will draw a white ball and then a red ball on two successive draws?

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9. If the first two statements below are true, is the last statement, called a conclusion, true or false? Why?

All baseball players have a baseball mitt.

Bill has a baseball mitt.

Therefore, Bill is most definitely a baseball player.

10. When the number 9 is multiplied by any of the digits 1 through 9, what characteristic do their products share in common besides being divisible by 9 and its factors?

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11. Bob is taller than Bill. Bill is shorter than Sharon. Sharon is taller than Sally. Sally is not the shortest. From the above information, how many different orders of tallest to shortest are possible?

12. The following is the name of a famous person written in a secret code. Can you decipher the code and discover the name of this person, who was known to unlock a few secrets of his/her own?

CNDGTV GKPUVGKP

13. A local chess tournament has five rounds of single elimination, including the championship match, with all matches played. How many chess players were originally entered?

14. In the following analogy, the third word is related to the fourth word in the same manner as the first word is related to the second. What is the word?

million : mega : : millionth : ?

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15. Below is an addition problem where the digits B and C each represent a number different from the numbers shown. What are the only two possible values for B and C?

$$\begin{array}{r} B\,4 \\ +C\,9 \\ \hline 1\,6\,3 \end{array}$$

16. Three snakes were out for a leisurely stroll when they ran into their worst enemy. See if you can assign a value for each letter to solve this alphametic (alphabet arithmetic problem). Each letter retains the same value within the problem, and that value must be different from any other letter. Zero may not begin a word.

HINT: Don't forget about "carryovers."

$$\begin{array}{r} A\,S\,P \\ A\,S\,P \\ A\,S\,P \\ +G\,A\,S\,P \\ \hline T\,R\,A\,P\,S \end{array}$$

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17. In the following letter replacement puzzle, there are only two cases where the letter "A" can be an odd number. What are they? Can S = 1? Are there any letters that can have only one number value in all cases? Why?

HINT: Make a chart of possible values for each letter. Each letter retains the same value within the problem, and that value must be different from any other letter. Zero may not begin a word. Don't forget about "carryovers."

```
  T O S S
+     T O
-------
  A N N E
```

18. Which one of the following words does not belong with the rest?

HINT: The answer is not the fact that one of the words ends in a vowel.

a. affordability

b. organization

c. confab

d. rehabilitation

e. abacus

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19. A bicyclist knows 2 different routes from Adams to Benton. From Benton to Clarion, the bicyclist knows 5 different routes. From Clarion to Denning, the bicyclist knows 4 different routes. How many different routes can the bicyclist take from Adams to Denning?

20. Can you find a common characteristic shared by the numbers 6 and 28 other than that they are both even numbers divisible by 2?

21. Eighteen is one half of 36; 36 is one half of 72. What is 144 divided by one-half?

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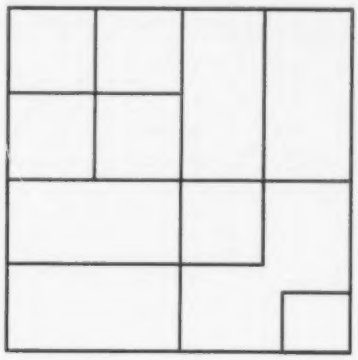
22. How many squares of any size can be found in the figure below?



23. Solve the following analogy:

HINT: It refers to animals who are active during the day.

night : nocturnal :: day : ?

24. Below are three grids. See if you can determine the logic to produce a fourth grid.

X		X	O
		X	O
O	O		
X			X

X	O		X
	O		
		X	X
X		O	O

X			X
		O	O
O	X		
O	X		X

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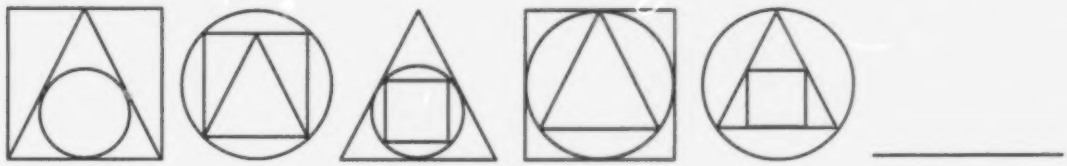
25. What does the next figure look like in the following sequence?



26. What is the next number in this series?

1 1 3 4 5 9 7 16 ?

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27. There are three numbers, A, B, and C, that have different values. Your boss has told you that the values of these numbers change daily. He needs a method that will print out these numbers from highest to lowest value as they appear at any moment. He's asked you to write a program for a machine that can decipher any commands written in English. The exception to this is that the machine will not accept blanket statements such as "print highest to lowest numbers." You have to use A, B, and C in your program. Can you write several statements that will allow your program to print out the numbers from highest to lowest value?

28. The prefix "in" can give the opposite meaning to a word, for example: sincere, insincere; fertile, infertile. Can you think of one or more words where the prefix "in" does not change the meaning?

29. How many digits do you need to change in the following addition problem to make the sum 240?

$$\begin{array}{r} 86 \\ 19 \\ +18 \\ \hline \end{array}$$

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30. Below are five sets of letters that are related in a way that is known to virtually everyone. Find the two missing letters in the sixth position. What are they?

HINT: Some people have been known to take months to solve this puzzle.

1. JD
2. FN
3. MO
4. AS
5. MA
6. ??

31. A game is played by two people where nine pennies are placed on a table. In alternating turns, each player picks up at least one penny, but not more than five pennies, on each turn. The player who picks up the last penny wins. How many pennies must the first player pick up on his/her first move to ensure victory?

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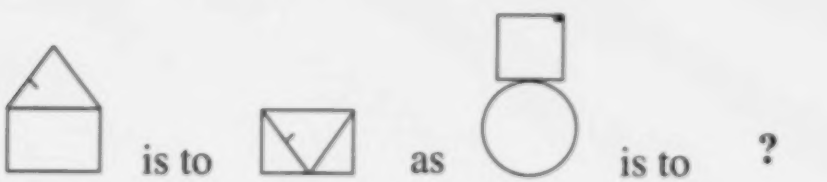
32. Six circles of the same size can intersect each other how many times (maximum)?

33. Below is an analogy where the first figure is related to the second figure as the third figure is related to the fourth. From the five given choices, can you determine which figure will complete the analogy?



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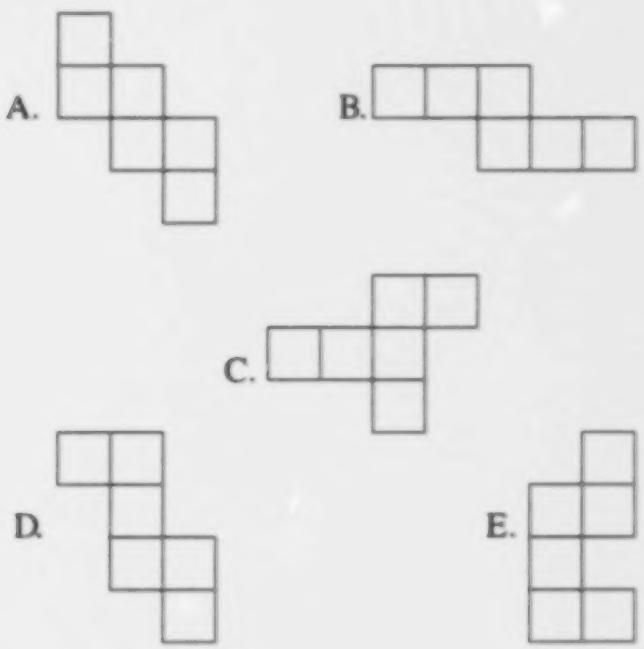
34. See if you can determine the logic to provide the missing number below.

35. Below are five versions of an unfolded cube. One of the five choices is impossible. Which one?



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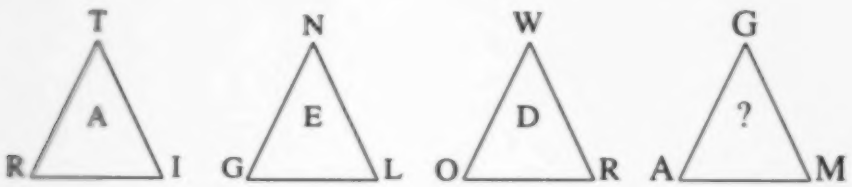
36. The letters arranged in and around the triangles below are positioned in a logical pattern that spells out a phrase. See if you can determine the pattern, supply the missing letter in the last triangle, and come up with the phrase.



37. What four-letter word can be placed in front of each of the following words to make four new words?

_____ SCRIPT _____ MAN _____ MARK _____ AGE

38. Three of the four animals below have a common characteristic. Pick the odd animal, and name the characteristic the other three share.

a. kangaroo

b. wombat

c. badger

d. opossum

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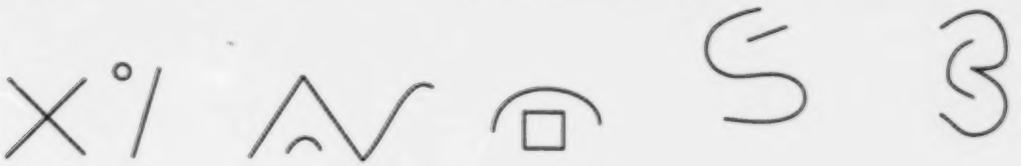
39. Four of the five figures below share a common characteristic that the fifth figure does not. Tell which one does not and why.



A. B. C. D. E.

40. What number is one-tenth of one-third of one-fourth of one-seventh of 1680?

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41. Below is a list of scores from a fictitious college football season. Based on the given scores only, if Hofstra were to play Michigan during this particular season, who would win and by how much?

Michigan 27, Nebraska 13

Hofstra 17, Northern Iowa 16

Nebraska 14, Oklahoma 10

Northern Iowa 24, Central Michigan 21

Oklahoma 10, Baylor 7

Central Michigan 28, Minnesota 24

Minnesota 35, Baylor 3

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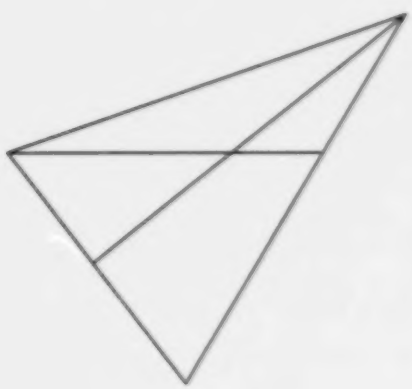
42. There may not be much call for the use of Roman numerals in our world, but they are fun to play with every now and then. Depending upon where the letters are located in relation to other letters, determining their values offers a challenge. See if you can match up the following Roman numerals with their correct values.

1. DL	a. 600
2. CD	b. 900
3. LD	c. 450
4. DC	d. 400
5. CM	e. 550
	f. 650
	g. 850
	h. 55

43. How many triangles of any size are in the figure below?



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44. A toaster that usually sells for $40 is on sale for 5% off. If 5% sales tax is then added to the sales price, what is the amount you would pay?

45. In the National Basketball Association, teams play four 12-minute quarters. One of the coaches has three guards on his team that are equal in ability. He wants all three guards to rotate so they all play the same number of minutes each game. Two guards (and only two guards) are always on the court during a game. How many minutes of playing time should each of the three guards receive?

46. Jamie's mother told him that in a certain drawer of the basement closet there were 12 cherry-flavored jawbreakers and 8 orange-flavored jawbreakers. Since there is no light in the closet, there is no way to know which flavored candy is being taken from the drawer. Jamie told his mom, "That's OK. I just want 2 orange-flavored jawbreakers, so I'll grab as many as I can and hope I get 2 orange candies." Jamie's mother said, "If you think about it, there is a minimum number of jawbreakers that you need to take in order to be assured of getting 2 orange jawbreakers." What is the least number of candies Jamie should take from the drawer to know for certain he has 2 orange-flavored jawbreakers?

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47. Many of the earlier puzzlists such as Sam Loyd and Henry Dudeney enjoyed creating conundrums to dazzle their readers. Most of the time these puzzles consisted of guessing a certain word or words from clues given in poem form. Below is a conundrum in which you are to determine two words from the given clues; the second word is to be derived from the first word.

HINT: The second word seems better when it's homemade.

To be here is unpleasant, that is for sure
Life is hidden and often a blur
But just add an "s" and things change so fast
Sweet replaces heat of the now recent past.

48. In many of Sam Loyd's conundrums he would give clues for each syllable, using the words first, second, third, etc. for each respective syllable. Below is a conundrum of this type. See if you can tie the three syllables together to come up with the word.

My first is quite common and one version of three
Choose the preposition, it moves things readily.

My second can be found on a higher degree,
Or you might think of home and her love eternally.

My third is on firm footing, all would agree.
In all there are ten that can be made wiggly.

My whole remains a puzzle for many you see.
Is it really a fruit or just another veggie?

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49. Here's a popular word game I've dubbed the "Triple Trickle Down." Below is a four-letter root word (PAST). By changing one letter at a time in the root word, fill in the three blanks to make the last word in each column. If you can reach the solution in less than three words, so much the better.

PAST

1. _____	1. _____	1. _____
2. _____	2. _____	2. _____
3. _____	3. _____	3. _____
4. NEAR	4. RICE	4. CURE

50. Try this Double Trickle Down:

MALT

1. _____	1. _____
2. _____	2. _____
3. _____	3. _____
4. FIST	4. BEAN

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51. Based upon the following information, how many Kelzons would Elise Olderog be given?

Patricia O'Connell was given 8 Kelzons.

Michaelangelo Durumbowski was given 12 Kelzons.

Bill Contori was given 5 1/2 Kelzons.

52. Anagrams are word puzzles that require the solver to rearrange letters to find the solution. Below is a phrase that, when unscrambled, gives the name of a world-famous person. The phrase also gives a clue to that person's identity. Who is it?

SMILES ON ACE

53. What is the following word when it is unscrambled?

TAKEPEAR

54. Alexa, her daughter Amanda, and Amanda's daughter Amy were all born on the same day of the same month. What are the chances of this happening?

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55. Let's take problem #54 one step further. Suppose an advertising company is looking for a grandmother, mother, and daughter of the same family, all with birthdays on December 25, as part of a promotion for Christmas. What are the chances that the grandmother, mother, and daughter were all born on Christmas Day?

56. Determine the missing digit in the number series below.

8 3 12 1 7 ? 16 5 14 9

57. What is the next number in the following series?

HINT: Check the differences between successive numbers.

3 2 11 14 27 34 ?

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58. Give yourself about two minutes for this puzzle. Can you come up with a quick method for finding the sum of the whole numbers 1 through 50 without adding the individual numbers together?

59. One cube of any size can be viewed in 24 different ways. There are 6 faces and each face can be turned 4 different ways; $6 \times 4 = 24$. If 2 cubes are fused together (one face of a cube is fused into one face of another cube), how many different ways are there to view this new solid object?

60. There are 100 people applying for a sales position where they will be required to sell both software and hardware for a computer company. Twelve of the applicants have no prior experience in sales. Sixty-four of the applicants have sold software and eighty of the applicants have sold hardware previously. How many of the applicants have experience in selling both software and hardware?

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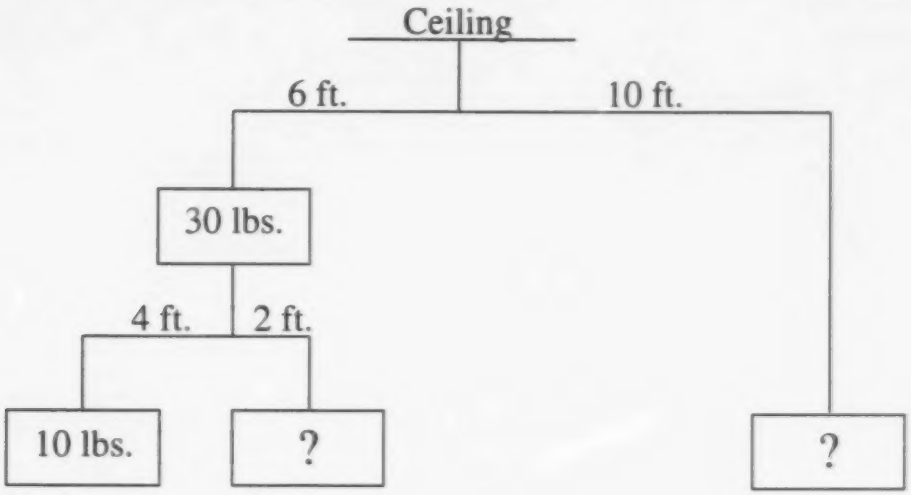
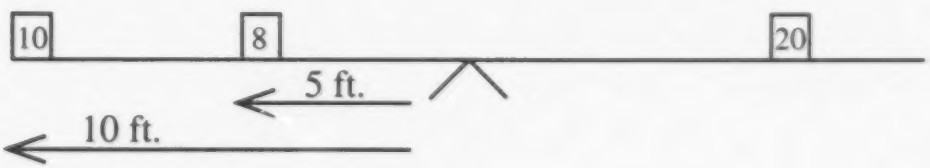
61. The following puzzle represents a mobile hanging from a ceiling. In order for the mobile to be perfectly balanced, as shown, put the correct weight in each box that has a question mark. (Disregard weight of supporting wires.)

HINT: $\text{Length}_1 \times \text{weight}_1 = \text{length}_2 \times \text{weight}_2$



62. Below is a "teeter-totter" with a 10-pound weight placed 10 feet from the fulcrum and an 8-pound weight placed 5 feet from the fulcrum. On the right side of the fulcrum is a 20-pound weight that needs to be placed in order to balance the weights on the left side. How many feet from the fulcrum should the 20-pound weight be placed?



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63. Below is a number series with a little different twist to it. See if you can determine a pattern to come up with the solutions for the two missing numbers.

HINT: Maybe there's more than one pattern.

8 2 7 4 6 6 5 8 4 10 ? ?

64. In the recent National Basketball Association lottery draft, the 11 teams that didn't make the playoffs were awarded an opportunity to make the first draft selection. To determine who would make the first selection, balls bearing the teams' names were randomly drawn from a bin. The team with the worst record in the regular season was given 11 balls in the bin, the team with the second-worst record was given 10 balls, etc. until the team with the best record received 1 ball. What were the chances that the team with the second-best record of the 11 teams would get to make the first draft pick?

65. The chance of picking the king of hearts from a shuffled deck of 52 playing cards is 1 out of 52. What are the chances that on two successive draws from a shuffled deck the first card chosen would be the king of hearts and the second card chosen would be the king of spades?

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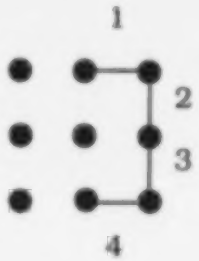
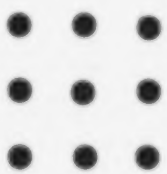
66. Below is a square of 9 dots. What is the maximum number of straight lines that you can draw between the dots without lifting your pencil and without crossing any lines you may have already drawn? Any line connecting two dots counts as one individual line. Use only horizontal and vertical lines.

Example:



The drawing above represents four lines.



67. In the land of Kooklatrope, a Kookla coin is equal in value to either 7 gold pieces or 12 silver pieces. If 40 Kookla coins are to be exchanged for both silver and gold pieces and there are only 119 gold pieces left at the exchange, how many silver pieces should you expect to receive with the 119 gold pieces?

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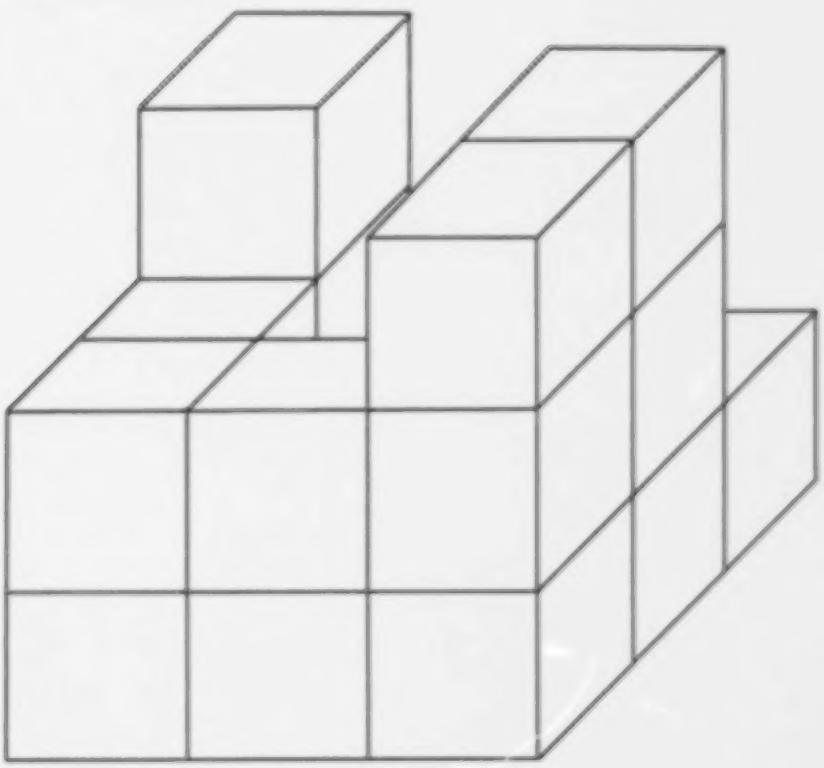
68. Fill in the missing numbers for each of the four question marks below. One of the question marks has more than one correct answer.

33	58	26	?
25	?	15	
?	17		
?			

69. How many individual cubes are in the configuration below? (All rows and columns run to completion unless you actually see them end.)



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70. If we count by 5s, starting with 1, the sequence begins like this: 1, 6, 11, 16, 21, 26, 31,... What is the 50th number in this sequence?

71. A box of candy can be equally divided (without cutting pieces) between 3, 4, or 5 people. What is the least number of pieces of candy the box could contain?

72. Imagine a cube 4" × 4" × 4" floating in space. Mentally cut the cube into 1-inch cubes. What is the maximum number of 1-inch cubes that are visible to an observer at any given moment?

HINT: You have the opportunity to view the cube from any angle.

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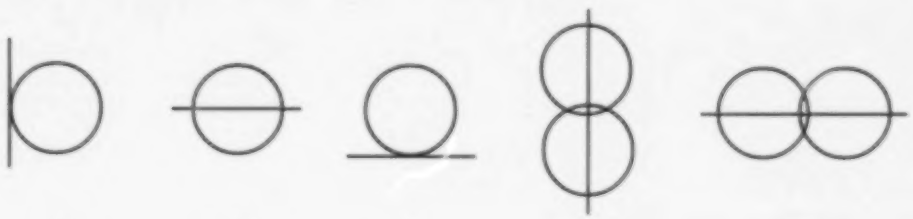
73. Which figure does not belong with the other four?

A. B. C. D. E.

74. Which figure does not belong with the other four?



A. B. C. D. E.

75. In the following series, fill in the question mark with the correct number.

120 60 20 5 1 ?

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76. What should the next figure in the sequence below look like?

77. The puzzle below is a form of syllogism. You are asked to consider the information given and determine whether there is a valid conclusion to be made from the statements. Is the conclusion below true or false?

Some pers are difs.

All difs are gorts.

Some gorts are tyns.

Therefore, some pers are definitely tyns.

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78. A husband and wife went shopping separately at different stores. When they came home, they both realized they had bought the same number of apples. The wife asked her husband how much he had paid for his apples, and he replied that they were marked 3 for 20 cents. The wife said that she had purchased her apples at 8 for 50 cents. Her husband said that he had spent only a nickel more than she had. How many apples had each bought?

79. Alex bought two different types of candles, one that burned for six hours and the other for eight. Both candles were exactly the same length. One evening, Alex lit both candles when his electricity went out during a thunderstorm. Sometime later, when one candle was exactly one-half the length of the other, the electricity came back on. Alex needed to reset the clocks that had stopped during the storm, but he didn't know how long the electricity had been out. Can you help Alex to figure out how long the candles have been burning?

HINT: The 6-hour candle burned 1/6 of its length in one hour or $x/6$ for x hours, leaving $(6 - x)/6$ of the candle.

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80. Below are ten analogies in which the relationship of the third member to the fourth member must be the same as the first member to the second member. You may not know the answer right away, but if you can establish the relationship between the first two members, you shouldn't have much trouble finding the answer with a little digging.

a. fly : insect :: spider : ?

b. trumpet : B-flat :: trombone : ?

c. light : dark :: transparent : ?

d. Redmen : Orangemen :: St. Johns University : ?

e. Ag : silver :: Au : ?

f. pint : gallon :: 1 : ?

g. centimeter : meter :: year : ?

h. snake : reptile :: wolf : ?

i. toward : away from :: windward : ?

j. straight : line :: curved : ?

81. Is there a conclusion that can be deduced from the following three statements? If so, what is it?

1. No baseball players are uncoordinated.
2. Bob steps on his wife's left foot when dancing.
3. No coordinated person ever steps on another person's foot.

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82. In a recent graduating high school class, 45 seniors were awarded $150,000 in grant money to be applied to their college education. The grants were divided into awards of $2,000 and $4,000. Each of the 45 students received one grant. How many students received $4,000 grants and how many received $2,000 grants?

83. A rap group consists of five people. The leader of the group always earns twice as much for a performance as each of the other four members of the group. For their most recent performance, the group received a check for $1,200. How much money does the leader of the group receive for his/her share?

84. Mary bought a clock and some wrapping paper for a total of $52. The clock costs $51 more than the wrapping paper. Quick now, how much did the wrapping paper cost?

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85. How many different ways can 5 children arrange themselves in a row of 5 seats?

86. Can you come up with at least 30 different words of any length that can be formed from the letters in the word READING?

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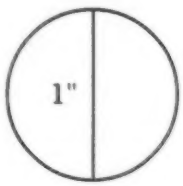
87. A small cube with a lid on top is 4" by 4" by 4" or 64 cubic inches in volume ($4 \times 4 \times 4 = 64$). How many ping pong balls with a diameter of one inch each can fit into the box so that the lid can be closed without smashing any of the balls? Remember, a diameter is the distance through the center of the ball.



88. Can you think of a situation in which the faster an object moves, the longer it will take to reach its destination?

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89. Which is larger, 2^{33} or $2^{32} + 2^{32}$?

90. If you know that ○ = 1, can you find the values for □ and △?

□ + □ + ○ + ○ = △ + △ + △ + ○ = △ + △ + □

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91. What is the value of F?

$$\begin{aligned} A + B &= Z \\ Z + P &= T \\ T + A &= F \\ B + P + F &= 20 \\ A &= 4 \end{aligned}$$

92. Abbie, Bill, Cara, and Doug are a freshman, a sophomore, a junior, and a senior, but not necessarily in that order. Doug told the freshman that Cara was a good tennis player. Abbie told Doug and the senior that the freshman was a good student. Doug didn't like the sophomore or Cara. In what grade is each person?

93. Below is a series puzzle that may seem a little strange at first. It was created in the galaxy of Sixtemus. Their number system isn't exactly like ours, but their logic is. See if you can determine the pattern and the number system to find the next number.

1 2 4 12 24 52 ?

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94. If our number system were based on 12 instead of 10, show a way that the first 12 numbers might be written.

95. Cherie and Cindy are Gwen's mother's son's wife's daughters. What relation is Gwen to Cindy and Cherie?

96. The following puzzle is one that features analytical reasoning. See if you can determine the relationships between the figures and words to find solutions to the two unknowns.

* * * = ?

SIZSOP = ?

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97. There are five books side by side on a shelf. Their colors are green, yellow, white, red, and blue. The following information is known about the order of the books:

1. The yellow book is between the blue and the red.
2. The green book is not first and the red is not last.
3. The white book is separated from the red book by two books.

What is the position of the green book?
If the green book is not next to the white book, what is the order, from first to last, of the five books?

98. The year 1961 is the last year in which you can turn the numbers upside down and backwards (rotate 180° counterclockwise) and they will read the same. When is the next year you'll be able to flip-flop the numbers of the year so they will read the same?

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99. What percent of Alpheds is Dexes?

Alpheds increased by 20% equal Brins.
Brins decreased by 50% equal Chons.
Chons increased by 30% equal Dexes.

100. If 1/2 of 6 were 4, what would 1/3 of 18 be?

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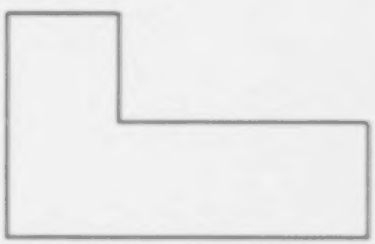
Answers

Some problems may have more than one correct answer. Any answer that can be justified with the given information is acceptable.

1. S
   These are the first letters of the days of the week with every other letter turned upside down, beginning with the first. The letter S looks the same when turned upside down.

2. 

3. Looking at the first two statements we know that one must be false. Bill's last name is either Johnson *or* Mason. Therefore, the third statement must be false since we know that two of the three statements are false. Since the third statement is false, the only choice left for Bob is to have the last name of Johnson, which means Bill's last name is Mason.

4. 44
   The difference between 2 and 4 is 2, between 4 and 8 is 4, between 8 and 14 is 6... The difference increases by two between each set of numbers. Therefore, the difference between 32 and the next number is 12:
   32 + 12 = 44.

5. These numbers correspond to the letters of the alphabet. A = 1, B = 2, C = 3,... The corresponding letters spell the word PUZZLES. S = 19.

6. Three dots
   Pivot the second die to the left so that the two dots are facing right: it will be in the same position as the first die. The five dots are opposite the two dots for a total of seven. Therefore, there must be three dots opposite the four dots.

7. Answers:
   a. Around and Around She Goes
   b. Time Is on My Side
   c. Deep in Thought
   d. Right through the Middle
   e. Stop in the Name of Love
   f. 90° in the Shade
   g. Root Canal
   h. There Are Two Sides to Every Story
   i. Dashed Hopes
   j. Dot Your "I's" and Cross Your "T's"
   k. Be on Time
   l. Light at the End of the Tunnel
   m. Reverse Order
   n. Rough around the Edges
   o. The Power of Positive Thinking
   p. Raised Consciousness

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8.

Here are the possible draws that could be made:

W1	W2
W1	R
W2	W1
W2	R
R	W1
R	W2

Out of six possible cases, a white followed by a red can be drawn in two cases. So the chances are two out of six or one out of three.

9. The answer is false. It is given that all baseball players have a mitt; it does not follow that all those who have a mitt are "most definitely" baseball players.

10.

9	9	9	9	9
×1	×2	×3	×4	×5
9	18	27	36	45

9	9	9	9
×6	×7	×8	×9
54	63	72	81

The sum of the digits in each product is 9.

11. There are three possible orders of tallest to shortest:

Bob	Sharon	Sharon
Sharon	Sally	Bob
Sally	Bob	Sally
Bill	Bill	Bill

12. ALBERT EINSTEIN
To decipher this code, you count back two letters in the alphabet from the letter in the code word and substitute the new letter (i.e., *a* substitutes for *c*, *l* substitutes for *n*, etc.).

13. 32
Each chess match requires 2 people. Since there are 5 rounds, the total number of chess players entered is $2 \times 2 \times 2 \times 2 \times 2$ or 32.

14. micro

15. Since a one is carried to B + C, making their total 16, B + C must equal 15. As 9 has already been used in the problem, B and C can only be 8 and 7. (We don't know which is which, but the problem doesn't ask for that information.)

16.

982
982
982
+7,982
10,928

17. Since *A* and *T* must be different numbers, there must be a carryover from the *O*. This means the letter *O* must always equal 9, which makes the letter *N* equal to zero. Once you know those two values, a chart of other possible values will be helpful in solving this puzzle.

S	T	E	A	
2	7	1	8	
3	6	2	7	..... CASE #1
4	5	3	6	
5	4	4	5	..... Impossible
6	3	5	4	
7	2	6	3	..... CASE #2
8	1	7	2	

From the chart, it can be seen that *A* can equal 7 or 3. *S* = 1 and *S* = 5 are not possible since those values would require the same

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number to be assigned to more than one letter.

18. B—organization. All the rest contain the letters *ab* together.

19. 40
If the bicyclist knows 2 different routes from Adams to Benton and 5 different routes from Benton to Clarion, then he or she can get to Clarion 10 different ways (5 × 2). From Clarion to Denning there are 4 different routes, so the bicyclist knows 10 × 4 or 40 different ways to cycle from Adams to Denning.

20. When the factors of each number (excluding the number itself) are added together, the sums of the factors equal their respective numbers—6 and 28. In mathematics, these are called "perfect numbers":
1 + 2 + 3 = 6;
1 + 2 + 4 + 7 + 14 = 28.

21. 288
Did I catch you on this? Anytime you divide by a fraction, you invert the fraction and multiply:
144 × 2/1 = 288.

22. 13

23. diurnal

24.

o	o		x
x	x		
		o	
x		o	x

Turn the first grid 90° clockwise to get the 2nd grid; then, turn 90° clockwise again to get the 3rd grid. One more turn 90° clockwise will give you the answer.

25. It is the only combination of the three figures that has not been previously used.

26. 9
Actually, this is two number series in one. Beginning with the first one and alternating with every other number, you have the series 1-3-5-7-9, the odd numbers. Beginning with the second one and alternating every other number, you have the series 1-4-9-16, the squares of the consecutive numbers beginning with 1.

27. Let the symbol > stand for "greater than."
If A > B and B > C, print A, B, C
If A > C and C > B, print A, C, B
If B > A and A > C, print B, A, C
If B > C and C > A, print B, C, A
If C > A and A > B, print C, A, B
If C > B and B > A, print C, B, A

28. flammable
inflammable

29. None. Turn the problem upside down.

$$\begin{array}{r} 81 \\ 61 \\ +98 \\ \hline \end{array}$$

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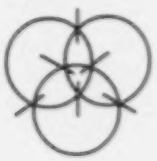
30. The "J" stands for January and the "D" for December. You work your way towards the middle of the year where you'll find J and J for June and July.

31. 3

32. This is a good puzzle to demonstrate the idea of using simpler, smaller examples to determine a pattern. Two circles can intersect each other twice:

Three circles can intersect each other 6 times:



If you were to draw four intersecting circles, you'd find the maximum number of intersections is 12.
Below is a table of the information we have:
2 circles — 2 intersections
$2 \times 1 = 2$
3 circles — 6 intersections
$3 \times 2 = 6$
4 circles — 12 intersections
$4 \times 3 = 12$
5 circles — ? $5 \times ? =$
6 circles — ? $6 \times ? =$

With each additional circle, we can see that the increase in intersections follows a pattern, multiplying the number of circles by the number one less than the number of circles. Five circles intersecting would then result in 5×4 or 20 maximum intersections, and six circles intersecting would result in 6×5 or 30 maximum intersections.

33. C
Imagine that the first figure is composed of a square and a triangle with a line extending from its left leg. In order to get the second figure, turn the triangle upside down and place it inside the square. Likewise for figure 3, turn the square upside down and place it inside the circle to complete the analogy.

34. The missing number is 6. The sum of the 2 numbers in the bottom half of the circle is one half the number in the top half of the circle.

35. E
There are 11 ways to unravel a cube, as shown below:

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36. The missing letter is *E*. Starting with the "T" in the first triangle and moving counterclockwise and then to the middle (in each triangle), the phrase spells "TRIANGLE WORD GAME."

37. POST

38. C—badger
The other three are marsupials and carry their young in an external pouch.

39. E
It is the only figure that doesn't contain a straight line.

40. 2
This sounds more complicated than it actually is. It looks like this:
$1/10 \times 1/3 \times 1/4 \times 1/7 \times 1680 =$
$1/840 \times 1680 = 2$

41. One of the keys to a puzzle such as this is to rearrange the given information into some semblance of order so that a logical comparison can be made. There are several ways to approach this. The following breakdown of games gives you one way to arrive at the solution.

Michigan 27, Nebraska 13
Nebraska 14, Oklahoma 10
Oklahoma 10, Baylor 7
Hofstra 17, Northern Iowa 16
Northern Iowa 24, Central Michigan 21
Central Michigan 28, Minnesota 24
Minnesota 35, Baylor 3

1. Minnesota beat Baylor by 32 points and Oklahoma beat Baylor by 3 points. If Minnesota were to play Oklahoma, they would win by 29 points.
2. Nebraska beat Oklahoma by 4 points and since Minnesota would have beaten Oklahoma by 29, they would beat Nebraska by 25 points.
3. Michigan beat Nebraska by 14 points, so Minnesota would beat Michigan by 25 – 14 or 11 points.

BUT

4. Central Michigan beat Minnesota by 4 points, so Central Michigan would beat Michigan by 15 points.
5. Northern Iowa beat Central Michigan by 3, so Northern Iowa would beat Michigan by 18 points.
6. Finally, Hofstra beat Northern Iowa by 1 point, so Hofstra would beat Michigan by 19 points.

42. 1 — e
2 — d
3 — c
4 — a
5 — b

43. 8

44. $39.90
If you said $40, don't feel bad. You fell into a trap that more than half the people who attempt this problem fall into. Let's take a look at why: 5% of $40.00 is $2.00. This $2.00 is then subtracted from the $40.00 to

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give $38.00. Now 5% of the sales price ($38.00) must be added back to give the correct purchase price: 5% of $38.00 is $1.90. Added to $38.00, this makes a total of $39.90.

45. 32 minutes
There are 48 total minutes for any given game. Since only 2 guards can play at one time, and there are 3 guards to play, each would play 2/3 of 48, or 32 minutes.

46. 14
Although highly unlikely, Jamie just might pull out the 12 cherry-flavored jawbreakers first. Then he would need to take 2 more to be certain that he has 2 orange-flavored jawbreakers.

47. desert
dessert

48. to-ma-toes

49. Below is one solution. If your solution differs from this or a friend's, that is acceptable as long as you followed the rules to arrive at the last word in any of the three columns.

1. PEST	1. PACT	1. PART
2. NEST	2. PACE	2. CART
3. NEAT	3. RACE	3. CARE
4. NEAR	4. RICE	4. CURE

50. Here's one way:

1. MAST	1. MELT
2. LAST	2. BELT
3. LIST	3. BEAT
4. FIST	4. BEAN

51. 6
Count the letters in each name and divide by 2 to determine the number of Kelzons each received.

52. Tennis star MONICA SELES

53. parakeet

54. 1 out of 133,225
Alexa could have been born on any of the 365 days of the year, but Amanda and Amy must have the same day and month as Alexa. Both Amanda and Amy have a 1 out of 365 chance of this happening. Since both Amanda's and Amy's birthdays have to be the same day and month, the products of their individual chances will give the correct solution: $1/365 \times 1/365 = 1/133{,}225$ or 1 out of 133,225 chances.

55. 1 out of 48,627,125
In this case, each of the 3 had a 1 out of 365 chance of being born on December 25. Multiplying all 3 chances together, you get $1/365 \times 1/365 \times 1/365$, which is 1 out of 48,627,125 chances.

56. 10
Starting with the two outermost numbers (8 and 9) and working toward the middle, each successive pair of numbers has 17 as its sum.

57. 51
In most number series, it is usually a good idea to look for a relationship initially by taking the differences between successive

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numbers to see if a pattern can be established. Don't give up right away if a pattern isn't established immediately.

3 2 11 14 27 34 ?
–1 9 3 13 7
10 –6 10 –6

As you can see in the third row, a pattern of 10, –6, 10, –6 is established. Therefore, working from the bottom up, you can see that 17 is the number that should be next to the 7, and 51 the number next to 34. You may have noticed that this sequence can also be determined by beginning with the number 1, squaring it and adding 2, squaring the number 2 (4) and subtracting 2, squaring 3 (9) and adding 2, squaring 4 (16) and subtracting 2, squaring 5 (25) and adding 2, etc.

58. One way to look at this problem is to see that the numbers 1 and 50 have a sum of 51, as do the numbers 2 and 49, 3 and 48, etc. There are 25 such pairs, and $51 \times 25 = 1{,}275$.

59. There would still be 24 different ways to view the fused cubes.

60. 56
Since 12 of the applicants have had no sales experience, we're dealing with 88 applicants who have some experience (100 – 12 = 88). Out of the 88 applicants, 64 of them have sold software, which means 24 of this group *haven't* sold software (88 – 64 = 24). Eighty applicants have sold hardware, which means out of the 88 applicants to be considered, 8 of these people *haven't* sold hardware. We now have 24 plus 8 people (out of 88) that could not have sold both, so 88 – (24 + 8) = 88 – 32 = 56 people left who have had experience selling both.

61. 20 lbs. left side; 36 lbs. right side
Consider that the mobile has a left side extending 6 ft. and a right side extending 10 ft. Not only must both sides be balanced, but the additional weights on the left side must be balanced with each other for the entire mobile to be balanced.

From the **hint** given, it can be seen that the two lower weights on the left side must be balanced with their two respective lengths taken into consideration:
4 ft. × 10 lbs. = 2 ft. × ? =
40 ft. lbs = 2 ft. × 20 lbs. The unknown weight on the left side is 20 lbs. This means that there are 30 + 10 + 20 = 60 lbs. total on the left side of the mobile which are supported through 6 ft. To find the unknown weight on the right side then, we have 60 lbs. × 6 ft. = 10 ft. × ? or 360 ft. lbs. = 10 ft. × 36 lbs. Therefore, 36 lbs. is the required weight for the right side.

62. 7 feet from the fulcrum
$(10 \times 10) + (8 \times 5) = 140$
$140 = 20 \times 7$

63. 3,12
Starting with the first number (8), look at every other number and see that a pattern of 8, 7, 6, 5, 4, ? is established. The first answer

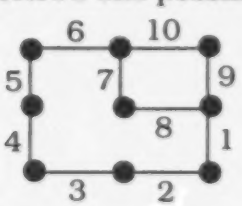
63

is 3. Starting with the second number (2), look at every other number and notice that 2, 4, 6, 8, 10, ? establishes a pattern where 12 is the answer to complete the puzzle.

64. 1 out of 33
The first thing that must be determined is how many total balls will be in the bin. That can be found by adding 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 = 66. Since the second-best team will be awarded 2 balls, their chances are 2 out of 66 or 1 out of 33 that one of their balls will be drawn on the first draw.

65. 1 out of 2,652
As we know from our example, the chance of drawing the king of hearts first is 1 out of 52. This leaves 51 remaining cards. Then the chance of drawing the king of spades on our second draw is 1 out of 51. Taking the two events together, we have $1/52 \times 1/51 = 1/2652$. The answer to the chance of drawing both kings is 1 out of 2,652.

66. 10
Here's one possible solution:



67. 276 silver pieces
If you were to exchange only for gold, it would require 40 × 7 or 280 gold pieces. But there are only 119 gold pieces, leaving you 161 gold pieces short. The value of silver coins to gold coins is in the ratio of 12 to 7.

$$\frac{12}{7} = \frac{x}{161}$$

Cross multiplying you get $7x = 1{,}932$ and $x = 276$.

68. 33 58 26 11 or 41
25 32 15
7 17
10
Each number directly below and between two other numbers is the *difference* of those two numbers.

69. 18

70. 246
One way to look at this is to make a chart showing the numbers in the sequence, their position, and the relationship of each number to its position.

number	1	6	11	16	21	26	31
position	1	2	3	4	5	6	7
difference	0	4	8	12	16	20	24

Notice that the difference for the first position is 0; after that, the difference between each respective number and its position increases by 4. To use this pattern to determine the number in any position, multiply by 5 the number of the position you are trying to find, then subtract 4.
50 × 5 = 250;
250 – 4 = 246.

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71. 60 pieces of candy
We're looking for the lowest whole number that can be divided by 3, 4, and 5 with the result being a whole number also. In other words, the lowest common denominator of 3, 4, and 5 will provide the correct answer.

72. 37
You can only see 37 of the 64 cubes at any one time. Remember, you can only count each cube once.

73. E
In each of the other figures, there is one line fewer in the middle than the number of sides the figure has. E has 5 lines and 5 sides.

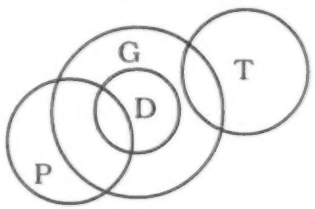
74. B
If just one circle appears in a figure, the line touches the circle only at the outer edge of the circle. If two interlocking circles appear in a figure, the line runs all the way through the figure. B has one circle with a line running through it instead of touching just the outer edge of the circle.

75. 1/6
The series progresses like this:
1/2 of 120 is 60
1/3 of 60 is 20
1/4 of 20 is 5
1/5 of 5 is 1
and the next would be
1/6 of 1, which is 1/6.

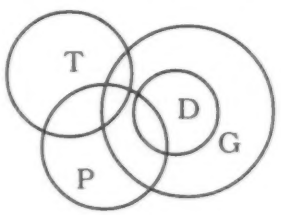
76.

The square moves clockwise around the inside of the figure and the circle moves counterclockwise.

77. The answer is false. Some pers *may be* tyns, but there is nothing to support the conclusion that some pers are definitely tyns. An easy way to set up this problem is through the use of Venn diagrams—circles used to represent each group. Depending upon the statements given, the circles may be linked or separate, or one circle may be placed inside another. Below are two ways to look at the relationships among the statements given in this problem.
Example 1:
where pers are *not* tyns



Example 2:
where some pers are tyns



As can be seen, some pers may be tyns, but it is not definite in this problem.

78. 12
The husband paid 6 2/3 cents for each apple while his wife paid

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6 1/4 cents for each of her apples: 2/3 – 1/4 = 8/12 – 3/12 = 5/12. The husband's apples were 5/12 of a cent more than his wife's. He spent a nickel more than she, so 5/12 of x = 5 cents, and x = 12 apples that were purchased by each.

79. 4.8 hours
From the hint, $(6 - x)/6$ of the 6-hour candle is left. Likewise, $(8 - x)/8$ of the second candle is left. But the 8-hour candle is twice as long as the 6-hour candle since it burns slower: $(8 - x)/8 = 2(6 - x)/6$. Cross multiplying, we have $6(8 - x) = 16(6 - x)$ or $48 - 6x = 96 - 16x$, which becomes $10x = 48$; therefore, x = 4.8 hours.

80. a. arachnid
b. E-flat
c. opaque
d. Syracuse University (University nicknames)
e. gold (from the periodic chart of the elements)
f. 8
g. century
h. mammal
i. leeward
j. arc

81. Bob is not a baseball player.

82. There are several ways to approach this problem. Here's one: A = students receiving $2,000 grants and B = students receiving $4,000 grants, so A + B = 45.
$2,000 × A + $4,000 × B = $150,000
Substituting, we get
$2,000(45 – B) + $4,000 × B = $150,000
$90,000 – $2,000B + $4,000B = $150,000
$2,000B = $60,000
B = 30
If there are 30 $4,000 grants, then that leaves 15 $2,000 grants.

83. $400
Let's say that each group member receives x dollars for his or her share. The leader then receives twice that amount or $2x$. Since there are 4 members, each receiving x dollars, and the leader receives $2x$, that makes a total of $6x$ dollars to be split among the group. They were paid $1,200 for their recent performance, so $6x$ = $1,200. If you divide each side of the equation by 6, you'll find x = $200. Therefore, the leader received twice this amount, or $400.

84. $.50
Remember, the clock is $51 more than the paper. If x is the cost of the paper, $x + 51$ is the cost of the clock.
$x + (x + 51) = 52$
$2x + 51 = 52$
$2x = 1$
$x = .50$

85. 120 different ways
The first child can choose any of the 5 seats; the second has 4 choices; the third 3 choices, etc.
$5 \times 4 \times 3 \times 2 \times 1 = 120$

86. There are more than 30 possible words—from short words like *a* and *in* to longer words such as *fringed* and *gander*.

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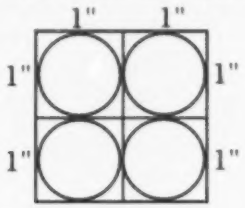
87. 64
The balls will fit in the following manner:



88. One example would be hitting a tennis ball. You can hit the ball directly across the net at a certain speed. If you hit the ball with more force and at a steeper angle, its speed will increase, but it will take longer to reach the other side of the tennis court.

89. They are the same. Try similar, easier examples using 2 as a base to check this; for example,
$2^5 = 2^4 + 2^4$
$32 = 16 + 16$

90. Let's rewrite this puzzle:
$2\square + 2 =$ $3\triangle + 1 =$ $2\triangle + \square$
1st part 2nd part 3rd part

Since there are now two different symbols ($\square$ and $\triangle$) contained in 3 different parts of the equation, we'll have to compare some of the equalities.
Comparing the first part and third part to solve for $\triangle$, we get

$$2\square + 2 = 2\triangle + \square$$
$$\square + 2 = 2\triangle \qquad (1)$$

Comparing the 2nd part and 3rd part to solve for $\triangle$, we get

$$3\triangle + 1 = 2\triangle + \square$$
$$\triangle = \square - 1 \qquad (2)$$

Now, let's take a look at equations (1) and (2):

$$\square + 2 = 2\triangle \qquad (1)$$
$$\square - 1 = \triangle \qquad (2)$$

If we multiply each member of equation (2) by 2, we'll get $2\square - 2 = 2\triangle$. Since we already know that $\square + 2 = 2\triangle$, then we know that

$$2\square - 2 = \square + 2$$
$$\square = 4$$

Since $\square = 4$, we substitute this back into our original puzzle and find $\triangle = 3$.

91. F = 14
Since it is known that A + B = Z, it then follows that A + B + P = T. We also know that T + A = F, so in the equation B + P + F = 20, we can replace F with T + A. The equation then becomes
B + P + T + A = 20 or
B + P + T = 16, since A = 4.
Rearranging the equations to solve for T, we then have

$$\begin{array}{r} 4 + B + P = T \\ +\ 16 - B - P = T \\ \hline 20 \qquad\quad = 2T \end{array}$$

Dividing both sides by 2 gives T = 10. Since we know A is 4 and T + A = F, F is equal to 4 + 10 or 14.

92. Abbie—sophomore
Bill—freshman
Cara—senior
Doug—junior

93. 144
Notice that no number is larger

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than 5. That's because these numbers are in the base 6 system instead of base 10. In base 10, the series would be 1, 2, 4, 8, 16, 32, 64. The answer 144 is not, of course, our base-10 144, but looks like this—

6^2	6^1	6^0
1	4	4
36	24	4

36 + 24 + 4 = 64 in base 10

94. It might look something like this: 1, 2, 3, 4, 5, 6, 7, 8, 9, =, $\underline{x}$, 10 (note that any symbols could be used to represent the old numbers 10 and 11). The first nine numbers could remain the same since they are single digits. But the old numbers we know as ten and eleven would each have to be changed to a single digit symbol. Our old number 12 (base 10) becomes 10. If you choose to call this number ten, then the symbols = and $\underline{x}$ would need new names, as would all the numbers that contain these two symbols.

95. She is their aunt.

96. Notice that the horizontal symbols both contain the letter "B" and it doesn't appear anywhere else. "B" stands for figures arranged horizontally. The letter "O" always appears with the * figure, so "O" stands for *. "I" therefore stands for the ∞ symbol. "P" stands for the number 4 and "Z" for 3. "S" appears only in the vertical arrangement of the symbols. This gives us all the information we need to know:

B = horizontal
O = *
I = ∞
P = 4
Z = 3
S = vertical
* * * = BOZ

SIZSOP =
∞
∞
∞
*
*
*
*

97. a. 5th position
b. white, blue, yellow, red, green

98. The year 6009

99. 78%
If Alpheds increases by 20%, it becomes 120% or 1.2, which is equal to Brins; 1.2 decreased by 50%, or 1/2, is equal to .6, which is Chons. Chons increased by 30% is .6 × .3 = .18 which, is equal to Dexes. Therefore, add .18 to .6.

$$\begin{array}{r} .60 \\ +.18 \\ \hline .78 \end{array}$$

The answer is 78%.

100. 8
The best way to view this problem is to set up proportions:
6/2 is to 4 as 18/3 is to x.
This can be written:
3 : 4 as 6 : x or

$$\frac{3}{4} = \frac{6}{x}$$

$$x = 8$$

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